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The implications of the thermal equilibrium assumption for surrounding-driven steady conduction within a saturated porous medium layer

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Abstract

The issue of local thermal equilibrium between a solid porous matrix and a saturating (stagnant) fluid under unidirectional steady heat diffusion is considered focusing on the effect of exterior-boundary thermal conditions. We demonstrate that in the case of exterior boundaries with uniform heat flux, i.e., when the heat per unit of phase area flowing through each phase is equal, thermal equilibrium will ensue only when the phases have the same thermal conductivity. When the phases are in thermal equilibrium at the exterior boundaries, then they will be in thermal equilibrium throughout the entire domain independently of the properties of each phase. In this case, the ratio of the heat flowing through each phase will be uniform within the layer and proportional to the thermal conductivity phase-ratio. With uniform heat flux at one end and thermal equilibrium at the other end of the domain, four independent criteria lead to thermal equilibrium. For a small fluid-to-solid volumetric heat transfer coefficient the medium reaches a state of maximum thermal nonequilibrium. A criterion for estimating the thermal equilibrium status within the layer is also derived. This criterion is important for the experimental determination of the effective thermal conductivity of a saturated porous layer. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

A	area [m^2]
Bi	Biot number
C_1, C_2	constants of integration
d	characteristic fluid–solid cell length [m]
h	convection heat transfer coefficient [W m^{-2} K^{-1}]
$\hbar$	volumetric heat transfer coefficient [W m^{-3} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
L	thickness of the porous layer [m]
$\mathbf{n}$	unit vector
q	rate of heat supply, equation (3) [W]
T	temperature [K]
V	volume [m^3]
x, y, z	Cartesian coordinates [m]
X	nondimensional coordinate, $X = xL^{-1}$.

Greek symbols

γ	surface porosity, equation (5)
Γ	ratio of nonequilibrium temperature gradient, $\Gamma = \delta_L \delta_0^{-1}$
δ	gradient of nonequilibrium temperature, equation (18) [K m^{-1}]
Δ	difference
θ	nondimensional temperature, $\theta = \lambda\tau\delta_0^{-1}$
κ	thermal conductivity phase ratio, equation (11), $\kappa = k_s k_f^{-1}$
λ	auxiliary parameter, equation (14) [m^{-1}]
Λ	nondimensional parameter, $\Lambda = \lambda L$
σ	porosity coefficient, equation (11)
Σ	summation
τ	nonequilibrium temperature, $\tau = (T_f - T_s)$ [K]
ϕ	volume porosity, equation (6).

Superscripts/subscripts

f	fluid phase
L	at $x = L$
n	index, $n = x, y, z$
s	solid phase (porous matrix)
0	at $x = 0$
$(\,)'$	spatial deviation
$(\,)''$	per unit of area
$(\,)'''$	per unit of volume
$(\,)^*$	microscopic.

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1. Introduction

Thermal diffusion is the simplest transport process in fully saturated porous media. It is also one of the most important for the implications it has on several engineering processes. For instance, heat diffusion determines (with the minimization of convection and radiation) the effectiveness of powder and fiber insulation materials and of desiccant porous materials [1–5]. It is also believed that maintaining a pure diffusion process within the mushy (porous) zone reduces macrosegregation during casting solidification [6]. Furthermore, diffusion dominated transport of heat and moisture can have a detrimental effect on important processes, like retarding the drying of fruits, grains and vegetables [7, 8]. Another important application is to phase-change within insulating porous matrices. In this case, the conduction regime is fundamental for predicting the initial phase-change development inside the matrix [9].

Only a very reduced number of studies published in the past years considered the fundamentals of heat diffusion through saturated porous media. We suggest that the two main factors are responsible for this lack of publications: (1) the complexity of the real process, and (2) the simplification of existing models.

The first factor has to do with the difficulty in characterizing the internal (pore-level) geometry of most porous media. Following precisely the internal structure of the solid matrix for studying the heat transport process taking place within a porous medium is a task beyond existing mathematical and computational tools. Therefore, the pore-level solution of the diffusion equation within a porous medium seems impractical at the moment.

The second factor follows from the necessity to develop suitable models for predicting, at least approximately, the heat diffusion process inside a saturated porous medium. The method of volume averaging [10–12], for instance, transforms the effects of the internal geometry of the solid matrix from a microscopic level (pore-level) into a macroscopic representative elementary volume level (rev-level). This transformation does not come free: heat transfer interaction at the solid–fluid interface within an rev must be accounted for and modeled. In trying to resolve this difficulty, the thermal equilibrium assumption, i.e., assuming negligible the difference between the volume-averaged solid and fluid temperatures within an rev, is usually invoked.

By applying the volume averaging technique and the thermal equilibrium assumption to the microscopic diffusion equation, one obtains (for isotropic media) the Laplace equation with an effective thermal conductivity parameter in place of the usual molecular thermal diffusivity (see p. 120 of Kaviany [13]). From this point on, the thermal equilibrium temperature distribution within the porous domain can be obtained from existing solutions of conduction heat transfer within homogeneous media by replacing the molecular conductivity with the effective thermal conductivity of the porous medium. The problem is then transferred from modeling the heat diffusion process per se to determining the effective thermal conductivity of saturated porous media, what is in itself an active research area [14–20].

Although the simplifications brought about by the volume averaging technique and the thermal equilibrium assumption are of practical significance, an important aspect seems to have been overlooked. In several applications the surrounding, or exterior boundary, drives the heat transfer process within the porous medium. The thermal interaction between the surrounding and the porous medium is represented by suitable boundary conditions. These boundary conditions must be consistent with the macroscopic equation, i.e., they must accord with the volume averaging approach and with the thermal equilibrium assumption.

Obviously, there might be situations in which the phases of the medium within a region near the exterior boundary are not in thermal equilibrium in which case a special treatment (e.g., two-temperature modeling) is necessary. Knowing the extent of this region is of interest because the thermal equilibrium model can still be used in the interior region of the domain where the thermal equilibrium assumption becomes valid.

We notice that the studies considering theoretical models for predicting the effective thermal conductivity of a porous medium (in line with the thermal equilibrium assumption) limit their scope to periodic structures for which the conditions at the exterior (macroscopic) boundaries of the porous domain do not interfere with the problem within the macroscopic domain, see for instance p. 82 of [21]: "In all this discussion, we have assumed that boundary effects at the exterior boundary of the entire porous domain do not interfere with the homogenization problem within the macroscopic domain." To validate these models experimentally, data must be collected within a region far enough from the boundaries where the thermal equilibrium assumption is valid. The size of this region—or distance from the boundaries—will certainly depend on the boundary conditions imposed to the medium.

There has been no attempt to clarify the implications of the exterior boundary conditions (used most frequently are the isothermal and isoflux conditions) on the thermal equilibrium assumption for a surrounding-driven heat conduction process through a porous medium layer. This is a very important and overlooked aspect because the idea of modeling the diffusion process with a single (average) Laplace equation and an effective thermal conductivity is meaningful only when the thermal equilibrium assumption is valid within the entire domain, including the exterior boundary region.

Another very curious aspect is that surrounding-driven steady heat conduction, in conjunction with the thermal equilibrium assumption, has been for years the preferred configuration for the experimental determination of the effective thermal conductivity of saturated porous media [22–25]. The effective thermal conductivity is usually calculated as the ratio between the measured heat flux and the temperature gradient along the solid phase computed using the temperatures measured at (or near) the boundaries of the porous domain. When the boundary region of the porous medium is at nonthermal equilibrium this procedure is invalid.

We note in passing that although referring to thermal diffusion through a saturated porous medium, when convection and radiation effects can be neglected (for criteria see [10]) the problem becomes analogous to that of heat conduction through composite solid materials [26–28].

Our analysis is distinct from the effect of nonuniform porosity near bounding surfaces on the effective thermal conductivity of a porous medium (see details in Kaviany [13], Section 3.7). In what follows we investigate the relationship between the exterior boundary conditions of a surrounding-driven heat conduction process and the thermal equilibrium assumption. For simplicity we restrict our reasoning to a plane, fully saturated, porous layer under steady conduction heat transfer. This simple domain has practical relevance because it is the preferred characteristic configuration for measuring the effective thermal conductivity of insulating porous layers and composites.

2. Model

Consider the following simplified steady heat diffusion equations for the fluid and solid phases of an isotropic porous medium, obtained from volume averaging (see, for instance, Kaviany [13], pp. 117–120, for details on deriving a more general equation)

$$0 = \phi k_{\rm f}\nabla^2 T_{\rm f} + k_{\rm f}\frac{1}{\rm V}\left[\left(\nabla\cdot\int_{A_{\rm fs}}\mathbf{n}_{\rm fs}T'_{\rm f}\,{\rm d}A\right) + \int_{A_{\rm fs}}\mathbf{n}_{\rm fs}\cdot\nabla T^*_{\rm f}{\rm d}A\right] \quad (1)$$

$$0 = (1-\phi)k_{\rm s}\nabla^2 T_{\rm s} + k_{\rm s}\frac{1}{\rm V}\left[\left(\nabla\cdot\int_{A_{\rm fs}}\mathbf{n}_{\rm sf}T'_{\rm s}\,{\rm d}A\right) + \int_{A_{\rm fs}}\mathbf{n}_{\rm sf}\cdot\nabla T^*_{\rm sf}\,{\rm d}A\right] \quad (2)$$

where ϕ is the porosity of the medium, $k_{\rm f}$ and $k_{\rm s}$ are the molecular thermal conductivities of each phase, $T_{\rm f}$ and $T_{\rm s}$ are the volume average temperatures, $T'_{\rm s}$ and $T'_{\rm f}$ are the spatial deviation temperatures, $T^*_{\rm s}$and $T^*_{\rm f}$are the local (microscopic) temperatures, $\mathbf{n}_{\rm sf}$ and $\mathbf{n}_{\rm fs}$ are unit vectors ($\mathbf{n}_{\rm sf} = -\mathbf{n}_{\rm fs}$) normal to $A_{\rm fs}$, the interphase surface area, and V is the volume of a representative elementary volume of the porous medium.

When considering the validity of the thermal equilibrium assumption, it is more convenient to replace the last term of equations (1) and (2) with a volumetric heat source (or sink) term, representing the energy exchange between fluid and solid phases. Written in terms of the volume average temperatures, we have

$$\phi k_{\rm f}\nabla^2 T_{\rm f} = \hbar(T_{\rm f} - T_{\rm s}) \quad (3)$$

$$(1-\phi)k_{\rm s}\nabla^2 T_{\rm s} = -\hbar(T_{\rm f} - T_{\rm s}). \quad (4)$$

This very peculiar term, namely $\hbar(T_{\rm f} - T_{\rm s})$, represents the macroscopic energy exchanged between fluid and solid within a volume V. The formal definition of $\hbar$ is then

$$\hbar = \frac{q'''_{\rm fs}}{(T_{\rm f} - T_{\rm s})} \quad (5)$$

where $q'''_{\rm fs}$ is the amount of energy per unit of time (power) exchanged between the fluid phase and the solid phase per unit of volume V. The heat transfer coefficient $\hbar$ should not be confused with the heat transfer coefficient h used in convection heat transfer analysis (this distinction was emphasized by Kaviany [13], p. 363, in the context of thermal nonequilibrium of convection heat transfer). For one reason $\hbar$ is defined independently of fluid motion, i.e., $\hbar$ is not necessarily zero when the fluid saturating the porous medium is stationary. Moreover, by definition the units of $\hbar$ are $\mathrm{W\,m^{-3}\,K^{-1}}$, and not $\mathrm{W\,m^{-2}\,K^{-1}}$.

Equations (3) and (4) can be recombined using the nonequilibrium temperature variable τ, defined as the difference between the fluid and solid temperatures, $\tau = (T_{\rm f} - T_{\rm s})$,

$$\nabla^2\tau = \hbar\left(\frac{1}{\phi k_{\rm f}} + \frac{1}{(1-\phi)k_{\rm s}}\right)\tau. \quad (6)$$

3. Unidirectional conduction

We now consider the simpler problem of conduction heat transfer through a porous, infinitely long, layer of thickness L. This can be treated as a one-dimensional steady conduction problem, from equation (6),

$$\frac{{\rm d}^2\tau}{{\rm d}x^2} = \lambda^2\tau \quad (7)$$

where

$$\lambda^2 = \hbar\left(\frac{1}{\phi k_{\rm f}} + \frac{1}{(1-\phi)k_{\rm s}}\right). \quad (8)$$

Equation (7) has the general solution

$$\tau = C_1\sinh(\lambda x) + C_2\cosh(\lambda x) \quad (9)$$

where the constants C_1 and C_2 are determined by imposing the boundary conditions in τ at $x = 0$ and at $x = L$.

3.1. *Case* 1: *heat flow boundary conditions*

The Neumann-type boundary conditions of equation (9) are obtained from the boundary conditions of each individual phase at $x = 0$ and L, namely

$$-\phi k_\mathrm{f}\frac{\mathrm{d}T_\mathrm{f}}{\mathrm{d}x}\bigg|_{0,L} = q''_\mathrm{f}|_{0,L} \quad \text{and} \quad -(1-\phi)k_\mathrm{s}\frac{\mathrm{d}T_\mathrm{s}}{\mathrm{d}x}\bigg|_{0,L} = q''_\mathrm{s}|_{0,L} \tag{10}$$

where the heat fluxes are per unit of total surface area A $(=A_\mathrm{f}+A_\mathrm{s})$. Conditions (10) can be combined and written as

$$\frac{\mathrm{d}(T_\mathrm{f}-T_\mathrm{s})}{\mathrm{d}x}\bigg|_{0,L} = \frac{\mathrm{d}\tau}{\mathrm{d}x}\bigg|_{0,L} = \delta_{0,L} \tag{11}$$

where

$$\delta_0 = \left[-\frac{q''_\mathrm{f}}{\phi k_\mathrm{f}} + \frac{q''_\mathrm{s}}{(1-\phi)k_\mathrm{s}}\right]_{x=0}$$

$$\text{and} \quad \delta_L = \left[-\frac{q''_\mathrm{f}}{\phi k_\mathrm{f}} + \frac{q''_\mathrm{s}}{(1-\phi)k_\mathrm{s}}\right]_{x=L}. \tag{12}$$

Observe that the fluid phase heat flux and the solid phase heat flux at $x = 0$ might not be necessarily the same as the heat fluxes at $x = L$. However, the total heat flux must be conserved in a steady regime, that is,

$$q'' = (q''_\mathrm{f}+q''_\mathrm{s})|_{x=0} = (q''_\mathrm{f}+q''_\mathrm{s})|_{x=L}. \tag{13}$$

The constants C_1 and C_2 of equation (9) are then determined by imposing the boundary conditions listed in equation (11). The final solution in this case is

$$\tau = \frac{1}{\tau}\{\delta_0\sinh(\lambda x)+[-\delta_0\coth(\lambda L) + \delta_L\,\mathrm{csch}(\lambda L)]\cosh(\lambda x)\}. \tag{14}$$

We can now consider the particular case of uniform heat flux condition at the boundaries, i.e., when the heat per unit of fluid-phase area is the same as the heat per unit of solid-phase area. In this case, $q'' = (q'''_\mathrm{f}/\phi) = [q''_\mathrm{s}/(1-\phi)]$ and equation (14) becomes

$$\tau = \frac{q''}{\lambda}\left[-\frac{1}{k_\mathrm{f}}+\frac{1}{k_\mathrm{s}}\right]\{\sinh(\lambda x)+[-\coth(\lambda L) + \mathrm{csch}(\lambda L)]\cosh(\lambda x)\}. \tag{15}$$

Therefore, the thermal equilibrium assumption, i.e., $\tau = 0$ everywhere, is inconsistent with the uniform heat flux condition unless the constraint $k_\mathrm{f} = k_\mathrm{s}$ is satisfied.

Consider now a situation in which one is capable of controlling the amount of energy (heat) being transferred to and from fluid and solid independently at the boundaries of the saturated porous medium. Consider further that the heat flow across the fluid is the same as the heat flow across the solid at $x = 0$ and L. With $q''_\mathrm{f} = q''_\mathrm{s} = (q''/2)$ from equation (13) in equation (12), and the result in equation (14), the difference between fluid and solid temperatures becomes

$$\tau = \frac{q''}{2\lambda}\left[-\frac{1}{\phi k_\mathrm{f}}+\frac{1}{(1-\phi)k_\mathrm{s}}\right]\{\sinh(\lambda x) + [-\coth(\lambda L)+\mathrm{csch}(\lambda L)]\cosh(\lambda x)\}. \tag{16}$$

In this case, the thermal equilibrium assumption $\tau = 0$ everywhere is inconsistent with the equally partitioned heat flow condition unless the constraint $\phi k_\mathrm{f} = (1-\phi)k_\mathrm{s}$ is satisfied. Therefore, the only way to obtain thermal equilibrium in the system is to compensate the thermal conductivity deviation between phases $k_\mathrm{f}/k_\mathrm{s}$ with an inverse phase-deviation of cross-section surface area $A_\mathrm{s}/A_\mathrm{f} = (1-\phi)/\phi$. Another interesting conclusion is that even when $k_\mathrm{f} = k_\mathrm{s}$ the thermal equilibrium assumption for equally partitioned heat flow conditions is not valid if $\phi \neq 0.5$. This is so because when the porosity is different than 0.5 the amount of heat going through one phase is different than the amount of heat going through the other phase.

Using equation (14) we can obtain, for the problem of conduction heat transfer across a layer with Neumann boundary conditions (not necessarily uniform heat flux or equal heat flow), general results for the deviation from thermal equilibrium. For simplicity we consider the dimensionless equivalent of equation (14),

$$\theta = \sinh(\Lambda X)+[\Gamma\,\mathrm{csch}(\Lambda-\coth(\Lambda)]\cosh(\Lambda X) \tag{17}$$

where the nondimensional quantities are: coordinate $X = x/L$, temperature $\theta = \lambda\tau/\delta_0$, and coefficient $\Lambda = \lambda L$. The parameter Λ is the ratio δ_L/δ_0, i.e., the ratio of nonequilibrium temperature gradients at L and 0, equations (11) and (12). We note in passing that the nondimensional parameter Γ is equivalent to $(Bi_\mathrm{s}+Bi_\mathrm{f})^{1/2}$, where Bi_s and Bi_f are the equivalent Biot numbers of solid and fluid phases, respectively, $(\hbar \mathrm{L})L/\phi k_\mathrm{f}$ and $(\hbar \mathrm{L})L/(1-\phi)k_\mathrm{s}$.

Figure 1 presents results of equation (17) for the case $\Gamma = 1.0$, a case equivalent to equal nonequilibrium temperature gradients at $x = 0$ and L. The graph presents the dimensionless nonequilibrium temperature distribution only along half of the domain, $0.5 \leqslant X \leqslant 1.0$, to help distinguish the curves for different Λ values. Observe, however, that the θ temperature distribution of equation (17) is an odd function of X in relation to $X = 0.5$, i.e., $\theta(X) = -\theta(1-X)$ for $0 \leqslant X \leqslant 1.0$. Therefore,the temperature distribution within $0 \leqslant X \leqslant 0.5$ is the inverted mirror image of Fig. 1.

We begin our discussion considering the case $\Lambda = 1.0$, shown in Fig. 1 with a thick line. This case reveals that θ increases almost linearly with X. Considering λ, L, and δ_0 as constants, let us say, the increase in θ is translated into an increase in the nonequilibrium temperature, i.e., the thermal nonequilibrium between the phases is enhanced along X. Observe that the middle of the domain, at $X = 0.5$, is the only location at which fluid and solid are in thermal equilibrium for any value of λ or δ_0.

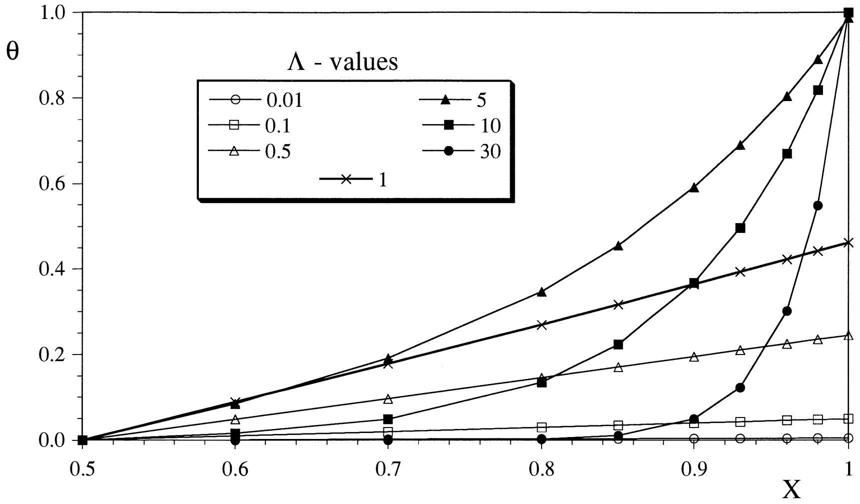

Fig. 1. Nondimensional nonequilibrium temperature distribution along half-domain for $\Gamma = 1.0$.

Decreasing the value of Λ to 0.5, 0.1, and 0.01, reveals a reduction in the θ variation with X. This observation, born out of Fig. 1, has to be taken cautiously. The fact is that θ is related to the nonequilibrium temperature via λ. If one considers L as being held constant, decreasing Λ decreases λ by the same factor. Therefore, the value of θ is automatically reduced even if the value of $(T_f - T_s)$, for constant δ_0, is maintained the same. In fact, this is exactly what seems to be happening: notice in Fig. 1 that $\theta(\Lambda_1)/\theta(\Lambda_2) \sim (\Lambda_1/\Lambda_2)$, where Λ_1 and Λ_2 are any two Λ values smaller than, or equal to, 1.0.

We can then conclude that for $\Lambda \leqslant 1.0$ the system rapidly tends to its maximum nonequilibrium condition. This means physically that the coefficient of energy transfer between fluid and solid h is so small as compared to the thermal conductivities that further decrease does not alter the temperature distribution within either solid or fluid phases. (Recall the definition equation (8) to recognize that small values of Λ can be obtained with small values of h, or in other words, media in which the transfer of energy between solid and fluid is increasingly hampered.) This conclusion has important practical implications: for an experimental configuration with equal nonequilibrium temperature gradients at the boundaries and $\Lambda \leqslant 1.0$, and where the temperature of only one phase is monitored, the response of the system to heat variation (for instance) is likely to be similar to the response from a system in thermal equilibrium when, in reality, the system is in its maximum thermal nonequilibrium state.

When the coefficient Λ increases beyond 1.0, the θ variation with X is further increased. However, the variation of θ with X is increasingly nonlinear: observe for instance the maximum variation in θ values along X to be less than 2.5 times when Λ increases by five times, from 1.0–5.0. This clearly indicates that the nonequilibrium temperature (or the difference in phase temperatures) is decreasing. This conclusion is confirmed by noticing the reduction of θ values when Λ increases further to 10, and then to 30. One should be careful with a further increase of Λ because, with the present boundary configuration, the energy equation (17) becomes singular: as Λ increases the gradient of θ at $X = L$ increases at the same rate (with $\Gamma = 1.0$, the boundary condition at $X = 1$ is $\mathrm{d}\theta/\mathrm{d}X = \Lambda$) but the values of θ along X tend to zero. These two remarks taken together indicate a singularity at $X = L$ when Λ tends to infinity.

The observation that $\Lambda \sim 1.0$ marks the threshold of asymptotic thermal nonequilibrium between the phases also for $\Gamma > 1.0$ and for $\Gamma < 1.0$. The main difference between the cases $\Gamma > 1.0$ (see Fig. 2 for $\Gamma = 1.02$, for instance) or $\Gamma < 1.0$ (see Fig. 3 for $\Gamma = 0.98$, for instance) and the case $\Gamma = 1.0$ is that the deviation from thermal equilibrium happens more abruptly as Λ decreases from 1.0. The symmetry around 0.5 is also lost when $\Gamma \neq 1.0$.

3.2. *Case 2: temperature boundary condition*

Considering the case of local thermal equilibrium at the boundaries $x = 0$ and $x = L$, i.e., $\tau_{0,L} = 0$, equation (9) gives $C_1 = C_2 = 0$. Therefore, when the phases are in thermal equilibrium at the boundaries the entire layer must be at thermal equilibrium, $\tau = 0$, for any x. This result together with equation (11) leads to $\delta_0 = \delta_L = 0$, which implies, via equation (12),

$$\left.\frac{q_f''}{q_s''}\right|_{x=0} = \left.\frac{q_f''}{q_s''}\right|_{x=L} = \frac{\phi k_f}{(1-\phi)k_s}. \tag{18}$$

Therefore, if the thermal equilibrium assumption at the boundaries of the domain is invoked, the heat flow

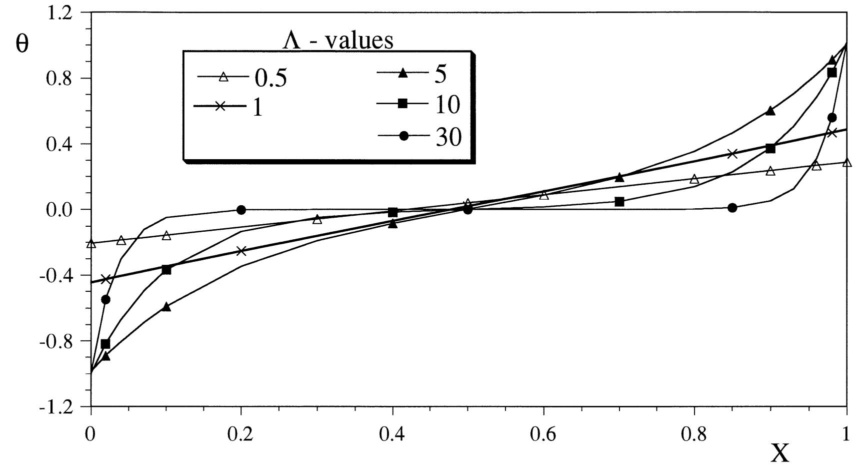

Fig. 2. Nondimensional nonequilibrium temperature distribution along X for $\Gamma = 1.02$.

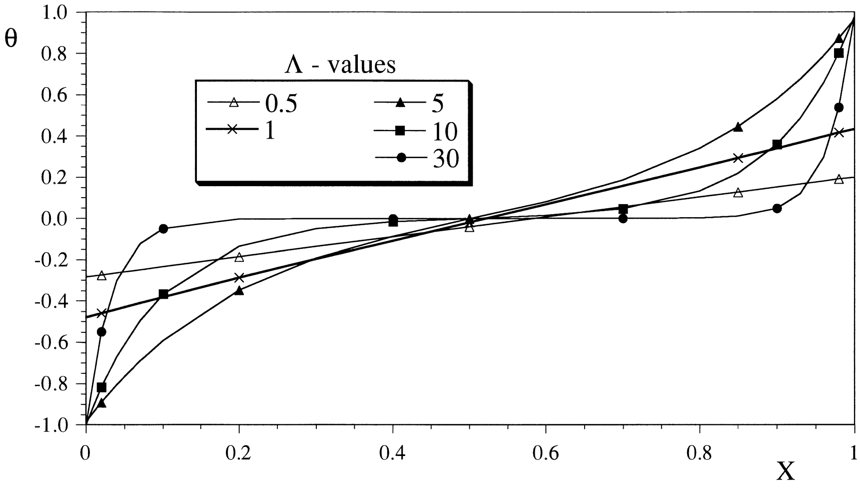

Fig. 3. Nondimensional nonequilibrium temperature distribution along X for $\Gamma = 0.98$.

through each phase must follow the relation given by equation (18). Coincidentally, equation (18) indicates that $\phi k_f = (1-\phi)k_s$ is a necessary condition for equally partitioned heat flow through the phases when the thermal equilibrium assumption is invoked at the boundaries, in agreement with the requirement found in case (1).

This case can be used in practice to check whether the thermal equilibrium assumption is valid by monitoring the difference of phase temperatures only at the boundaries of the porous layer.

3.3. *Case* 3: *hybrid boundary conditions*

Consider now equation (7) with uniform heat flux $q'' = q''_f/\phi = q''_s/(1-\phi)$ at $x = 0$, and equilibrium temperature at $x = L$, respectively, $(\mathrm{d}\tau/\mathrm{d}x)_0 = \delta$ and $\tau_L = 0$, where

$$\delta = q''\left[-\frac{1}{k_f}+\frac{1}{k_s}\right]. \tag{19}$$

This case is perhaps the most realistic (practical) model of the experimental configuration used for measuring the effective thermal conductivity of a saturated porous medium: uniform heating at one end (with electric resistance, for instance) and isothermal cooling at the other end (e.g., with two-phase convective cooling). The general solution is then

$$\tau = \frac{\delta}{\lambda}[\sinh(\lambda x) - \tanh(\lambda L)\cosh(\lambda x)]. \tag{20}$$

The deviation from thermal equilibrium in this case is identically zero everywhere in the domain when $\delta = 0$ or when λ tends to infinity. From equation (19) the first condition translates into $k_f = k_s$. The second condition is satisfied, from equation (8), when $\hbar$ tends to infinity, i.e., when the heat transfer between phases is so efficient that the phases cannot sustain a nonequilibrium condition, or when either ϕk_f or $(1-\phi)k_s$ tends to zero, that is, when either phase is adiabatic or nonexistent within the domain.

The nondimensional form of equation (20) is

$$\theta = \sinh(\Lambda X) - \tanh(\Lambda)\cosh(\Lambda X). \tag{21}$$

Figure 4 presents the nonequilibrium temperature distribution in X, for several values of Λ. In contrast with Fig. 1, the nonequilibrium temperature distribution is now plotted throughout the entire domain X because the right-hand side of equation (21) is no longer an odd function of X. The combining effects of equally partitioned heat flux condition at $X = 0$ and equilibrium temperatures at $X = 1$ are evident.

The observations in the paragraphs following Fig. 1 are relevant to Fig. 4. The case $\Lambda = 1$ of Fig. 4 reveals that θ decreases almost linearly with increasing X. Assuming constant λ and δ, the decrease in θ is translated into a decrease in the nonequilibrium temperature τ, i.e., the nonequilibrium disappears when approaching $X = 1$, as expected.

Decreasing the value of Λ to 0.5, 0.1, and 0.01, reveals a clear reduction in the θ variation with X. Notice in Fig. 4 that $\theta(\Lambda_1)/\theta(\Lambda_2) \sim (\Lambda_1/\Lambda_2)$, where Λ_1 and Λ_2 are any two Λ values $\leqslant 1.0$. We can then conclude, again, that for $\Lambda \leqslant 1.0$ the system is in maximum nonequilibrium condition. When the coefficient Λ increases beyond 1.0, the θ variation with X is further increased. However, the variation of θ with X is increasingly nonlinear indicating that the nonequilibrium temperature (or the difference in phase temperatures) is decreasing.

These observations have an important practical implication: for an experimental configuration with equal heat flux partition between the phases at one end, thermal equilibrium between the phases at the other end, and with $\Lambda \leqslant 1.0$, most of the system is likely to be in thermal nonequilibrium. Because of the difficulty in monitoring the average temperatures of both phases, one could use the criterion $\Lambda \gg 1.0$ to signal when most of the system is in thermal equilibrium (see for $\Lambda = 30$, in Fig. 4, only 20% of the entire length of the layer would be in nonthermal equilibrium). In dimensional form, the criterion translates into:

$$L \gg \left[\hbar\left(\frac{1}{\phi k_f} + \frac{1}{(1-\phi)k_s}\right)\right]^{1/2}. \tag{22}$$

Notice that criterion (22) is also valid when determining the thermal equilibrium condition for the general configuration of Case 1.

Unfortunately, there are not many models available in the literature for estimating $\hbar$. As an example we cite Quintard and Whitaker's [30] suggested relation:

$$\hbar = \phi k_f \frac{40\sigma\kappa}{d^2} \frac{(\sigma^2+\sigma+1)}{(1+5\kappa)+\sigma(2+\kappa)+(\sigma^4+2\sigma^3+2\sigma^2)(1-\kappa)} \tag{23}$$

where d is the characteristic length of a fluid and solid (sphere) cell, $\sigma = (1-\phi)^{1/3}$ and $\kappa = k_s/k_f$. Although derived considering a periodic array of spheres, relation (23) can provide a reasonably good estimate of $\hbar$ even in regions near the boundaries when the rev of the medium is small.

We point out that although in most practical configurations the value of $\hbar$ is relatively large, making the

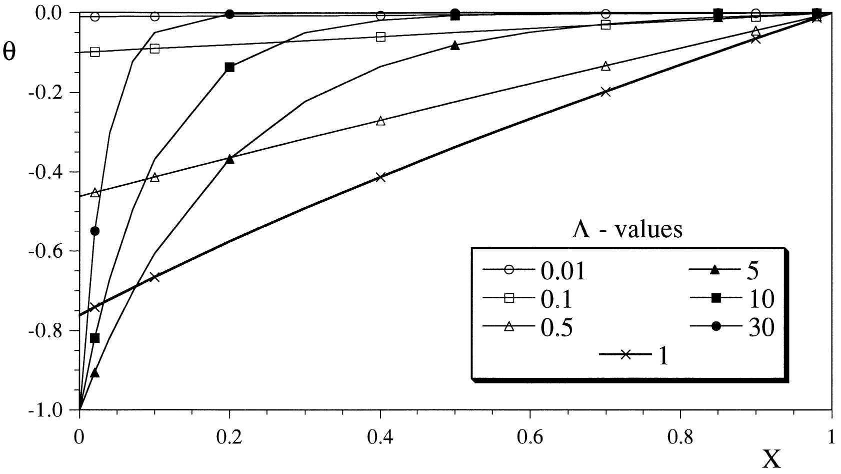

Fig. 4. Nondimensional nonequilibrium temperature distribution along X: equal heat flow at $X = 0$, thermal equilibrium at $X = 1$.

maximum thermal nonequilibrium state unlikely to occur, the difference between solid and fluid temperatures at the uniform heat flux boundary can be very large. For instance, consider a 1.0 m porous layer consisting of copper spheres ($\phi = 0.4$, $d = 0.02$ m, $k_s = 386$ $\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$) saturated with water ($k_f = 0.61$ $\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$) and heated at 15 $\mathrm{kW\,m^{-2}}$ from one end and cooled isothermally at the other. The thermal nonequilibrium, i.e., the difference between solid and fluid temperatures, at the heated end is approximately 75°C. This temperature difference affects the calculation of the effective thermal conductivity of the medium based on the difference of solid temperatures measured at the two boundaries of the medium. Therefore, it is essential that the temperature measurement be done away from the heated boundary.

This kind of departure from the thermal equilibrium condition at the heated boundary can be responsible for the strange behavior and the scatter observed on plots of experimental effective thermal conductivity data [11].

Before proceeding we should consider the statement made by Quintard and Whitaker [30] on page 2784: "... local thermal equilibrium is always valid for steady 1D heat conduction." This conclusion was drawn, following the average Laplace equation, from the observation that the temperature gradient must be constant in one-dimension steady conduction. However, we have demonstrated that the average Laplace equation is significant (valid) only when thermal equilibrium exists and that thermal equilibrium is closely tied to the boundary conditions imposed by the surroundings of the saturated porous material. This aspect was not considered by Quintard and Whitaker, even though they insinuated the limitations of their approach by writing on page 2784: "... information about a field ... was best obtained by an examination of the ... boundary conditions .... However, the estimate ... is based only on the governing differential equation and thus must be used with some care."

4. Summary and conclusions

We have examined the issue of boundary conditions and thermal equilibrium on a fully saturated porous medium considering the simple configuration of steady conduction through a porous layer.

Our analysis has demonstrated that when a uniform heat flux condition (same heat flow per unit of phase area through each phase) is imposed at both boundaries of the layer, the phases will be in thermal equilibrium only if $k_f = k_s$. In the case of equal heat flow through each phase, the saturated porous layer will be in thermal equilibrium only if $\phi k_f = (1-\phi)k_s$.

Furthermore, if the boundary temperature of the phases are equal (i.e., thermal equilibrium at the external boundaries), the entire saturated porous medium will be in thermal equilibrium, and the ratio of fluid heat flux to solid heat flux is unique and easily predicted.

The case of hybrid boundary conditions considering uniform heat flux at one end and thermal equilibrium at the other is also investigated. This configuration yields four conditions for thermal equilibrium, namely $\phi k_f = (1-\phi)k_s$, $\hbar \to \infty$, $(k_f, k_s) \to 0$, or when $\phi \to (0, 1)$. The condition $\Lambda \leqslant 1.0$ limits the nonequilibrium temperature to its maximum state; a further decrease in Λ does not alter the temperature distribution within each phase along the porous layer.

We have shown also that although for most practical situations thermal equilibrium will prevail in most of the layer, the thermal nonequilibrium at the heated boundary can drastically affect the measurement of the effective thermal conductivity of the medium. Care should be exercised by measuring the phase temperature, within the layer, at a sufficient distance from the heated boundary to satisfy the thermal equilibrium condition. A criterion to estimate this distance has also been presented. Existing effective thermal conductivity data should be reanalyzed in light of these new observations.

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